1. A medical researcher is studying the number of hours, T, a patient stays in hospital following a particular operation.

The histogram on the page opposite summarises the results for a random sample of 90 patients.

(a) Use the histogram to estimate $P(10 < T < 30)$

(2)

For these 90 patients the time spent in hospital following the operation had

- a mean of 14.9 hours
- a standard deviation of 9.3 hours

Tomas suggests that T can be modelled by $N(14.9, 9.3^2)$

(b) With reference to the histogram, state, giving a reason, whether or not Tomas' model could be suitable.

(1)

Xiang suggests that the frequency polygon based on this histogram could be modelled by a curve with equation

$$y = kxe^{-x} \quad 0 \leqslant x \leqslant 4$$

where

- x is measured in **tens of hours**
- k is a constant

(c) Use algebraic integration to show that

$$\int_0^n xe^{-x}\, dx = 1 - (n + 1)e^{-n}$$

(4)

(d) Show that, for Xiang's model, $k = 99$ to the nearest integer.

(3)

(e) Estimate $P(10 < T < 30)$ using

(i) Tomas' model of $T \sim N(14.9, 9.3^2)$

(1)

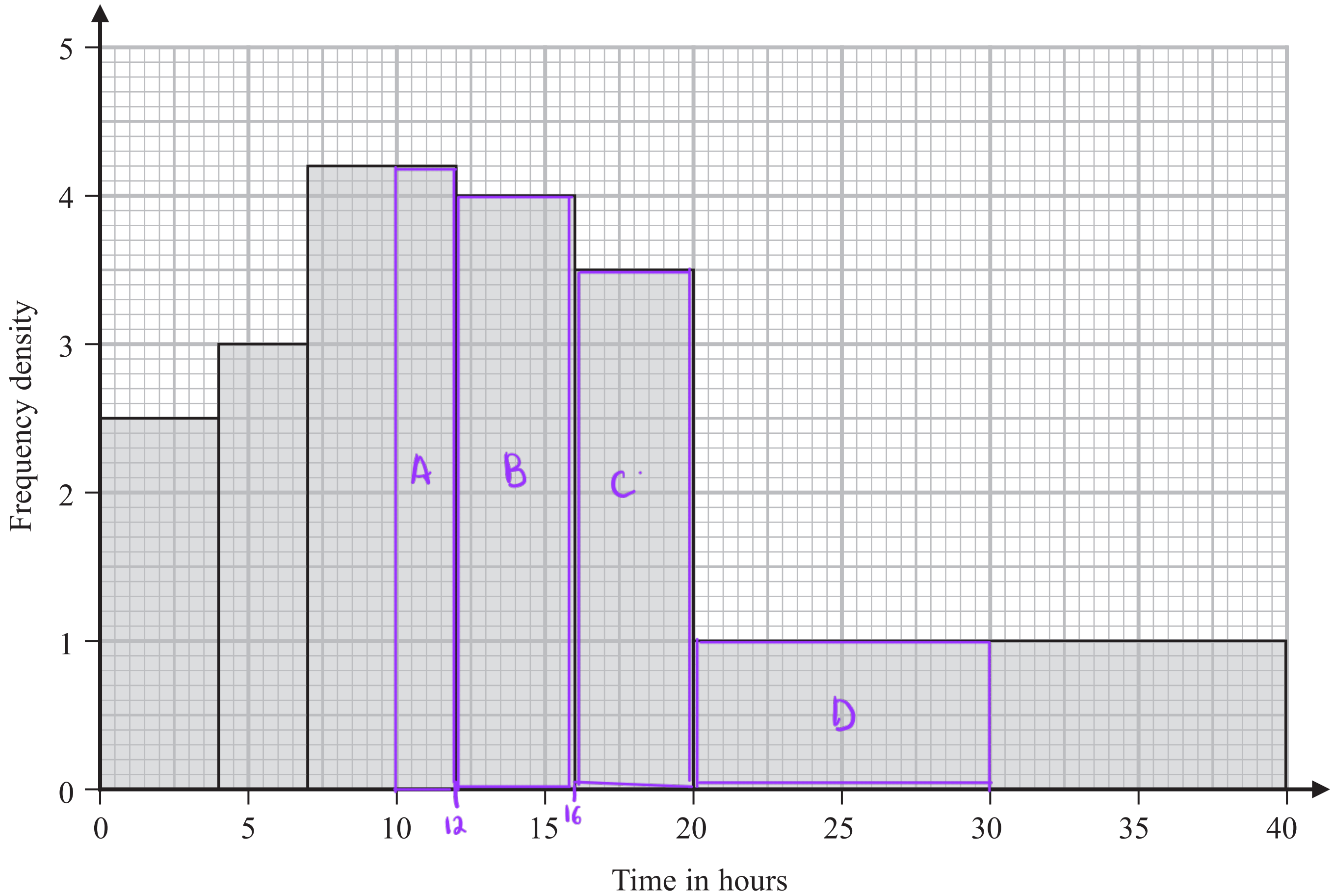
(ii) Xiang's curve with equation $y = 99xe^{-x}$ and the answer to part (c)

(2)

The researcher decides to use Xiang's curve to model $P(a < T < b)$

(f) State one limitation of Xiang's model.

(1)



a) $P(10 < T < 30) = P(A) + P(B) + P(C) + P(D)$

$= \dfrac{(2 \times 4.2) + (4 \times 4) + (4 \times 3.5) + (16 \times 1)}{90}$ (1)

$= \dfrac{8.4 + 16 + 14 + 10}{90}$

$= \dfrac{48.4}{90}$

$= 0.5377... = 0.54$ (2 s.f.) (1)

(b) It does not look suitable because a normal distribution is symmetrical and the histogram is not. (1)

c) $u = x \qquad v = -e^{-x}$

$u' = 1 \qquad v' = e^{-x}$ 1

$u \times v - \int vu'$

$\therefore \int xe^{-x}\,dx = -xe^{-x} - \int (-e^{-x})\,dx$ 1

$= -xe^{-x} + \int e^{-x}\,dx$

$= -xe^{-x} - e^{-x}$

$\therefore \int_0^n xe^{-x}\,dx = \left[-xe^{-x} - e^{-x}\right]_0^n$

$= (-ne^{-n} - e^{-n}) - (-e^{0})$ 1

$= -ne^{-n} - e^{-n} + 1$

$= 1 - (n+1)e^{-n}$ 1 (shown)

d) Area under frequency polygon = 90 when $n = 4$

$\therefore k\int xe^{-x}\,dx = 90$ 1

$\therefore k\{1 - (n+1)e^{-n}\} = 90$

when $n = 4$: $k(1 - 5e^{-4}) = 90$ 1

$k = \dfrac{90}{1 - 5e^{-4}}$

$k = 99.07\ldots$ 1

$\therefore k \approx 99$

e) (i) $T \sim N(14.9, 9.3^2)$

$P(10 < T < 30) = 0.6486.... \approx 0.649$ (3s.f.) 1

(ii) No° of patients

$\approx \int_1^3 99xe^{-x}\,dx$

$= 99\{(1-4e^{-3}) - (1-2e^{-1})\}$

$\approx 53.1....$ 1

$\therefore$ Probability $= \dfrac{53.1....}{90}$

$= 0.590...$ 1

(f) Some patients might stay longer than 40 hours. 1